

The signalman against the glut and gap theorists

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ABSTRACT: Radical glut and gap theorists deny—in opposite ways—that the liar sentence has exactly one of the two values *true* and *not true*. I describe a scenario where a signalman finds himself in a situation analogous to the liar paradox: if he lights a fire at a certain time, that is analogous to the liar being true, and if he does not, that is analogous to the liar not being true. It is obvious that he must make exactly one of those states of affairs come about. It is argued that there are no relevant differences between the liar and the signalman’s dilemma, implying that the glut and gap theorists are wrong about the former. A further point is that whether or not the liar is true/the signalman lights the fire, language/the signalman is misleading relative to the conditions under which the liar/the fire “ought” to be true/lit.

1 THE SIGNALMAN AND THE LIAR

Scenario 1: λ is the English sentence “ λ is not true”.

Claim about scenario 1: λ is either true or not true, and not both (whether it is one or the other, the truth value it has is misleading: it doesn’t represent the facts as they are to the language users).

This claim is controversial. It is denied by *glut theorists* or *dialetheists*, who think that λ is both true and not true, and by *gap theorists*, according to whom it is neither true nor not true.¹

Scenario 2: Once upon a time, in a kingdom far, far away, an old man lived on top of a mountain. He was known as “Kuvata” and he was a signalman. He had been sent to live on the mountain by the king so that

¹Actually, the gap theorists I have in mind would never allow themselves to say “ λ is neither true nor not true” because that claim itself is not part of their theory. They are the theorists that one might argue are obligated to be quietists on the issue of the truth value of λ , on pain of hypocrisy. However, those who have written about it have suggested ways out that allow them to write about it: for example, that they can *deny* λ and also *deny* the negation of λ , or they might say that they do not “accept that there is a truth of the matter” (Field 2008, 70). And actually, the glut theorists I have in mind might very well accept that λ is not both true and not true. It is just that they would also accept that λ is both true and not true—see, e.g., Priest (2006).

he could make observations from his high vantage point and convey them to the king's court. He used a system as old as the kingdom itself to send the signals: light a fire at a specified time, or refrain from doing so. Each winter's solstice, a foot messenger painstakingly climbed up the mountain with a parchment containing instructions from the court. One year, they read as follows: "Each day in the coming year, you must look for enemies crossing the sea, and if they do—and only if they do—you must light a fire at sundown. You must also watch out for storms approaching from the west, and if one does—and only if one does—you must light a fire one sandglass after sundown." The parchment concluded with an instruction to pass on a signal from another signalman, whose fires were within sight of Kuvata's mountain, but not of the palace: "Finally, you must keep an eye on whether the signalman on Mount Íslensku lights a fire two sandglasses after sundown, and if he does—and only if he does—you too must light a fire two sandglasses after sundown." That year, like every previous year, Kuvata lit a fire exactly when he ought to. Doing so was his sworn duty and a matter of honour to every member of the Signalmen's Guild.

When the next year's instructions arrived, they were brief: "Each day in the coming year, if the signalman on Mount Suomalainen does not light a fire at sundown—and only if he does not—you must light a fire at sundown." But Mount Suomalainen was Kuvata's own mountain!

Claim about scenario 2: Kuvata either lights the fire at sundown or he does not, and not both (whether he does it or not, his action is misleading: it doesn't represent the facts as they are to the court).

This claim is completely uncontroversial. Anyone would agree that in this scenario the signalman cannot defy the law that any state of affairs either obtains or fails to obtain.

What could possibly justify diverging opinions about the two claims? If Kuvata does not light the fire, he (in a certain sense) ought to light it, and if he does light it, he ought not to. The ought-to-light condition is satisfied if and only if the fire is not lit. The two cannot match. The signalman cannot represent the facts as they are. Similarly, if λ is not true, it (in a certain sense) ought to be true, and if it is true, it ought not to be. The ought-to-be-true condition of λ is satisfied if and only if λ is not true. The two cannot match. The sentence cannot represent the facts as they are.

The motivation for glut and gap theory comes down to just one thing, namely the assumption that the set of all true sentences must be a perfect representation of the facts of the world. Under this assumption, λ cannot be true only because then the truth of " λ is not true" would misrepresent the fact that it is true and therefore "ought" to be not true, and vice versa for the option of λ being not true only. This assumption of the ability of language to represent perfectly therefore clashes with the principle that any state of

affairs either obtains or fails to obtain and not both (specifically the state of affairs of λ being true). Glut and gap theorists believe that the principle of perfection of language must trump this basic metaphysical principle.

But why would language be able to do better than a signalman who is fanatical about representing the facts as they are? Language is, after all, created by humans and there is no reason to think that we, in this regard, are more powerful collectively than the signalman is individually. Is it because *propositions* are the real truth-bearers, so that the *real* language is not created by humans, and, being abstract and disembodied, the *real* language of propositions is divine and therefore able to do what is impossible for mere mortals? Making that kind of move is certainly a well-tested method for forcing a desired element of perfection into a worldview where it otherwise seems lacking.

Is the difference that English is semantically closed, and that semantic closure can force states of affairs to both obtain and fail to obtain, or neither obtain nor fail to obtain? The entire communication system in scenario 2, consisting of the court sending Kuvata instructions and him trying to follow them, is also semantically closed (if English is), as the court can use the full resources of English to convey a criterion for signalling to the signalman.

Is the difference that English is compositional? If Kuvata didn't receive individual instructions for a finite number of instants of time, but instead received an instruction manual containing recursive rules according to which he could deduce what was expected of him at each instant of time during the day by applying the rules to, say, a number that represents the instant of time in some chronometric system; and it was a result of *those* rules that the ought-to-light condition for Kuvata at sundown is that Kuvata does not light a fire at sundown, would he then both light the fire and not light the fire (or neither light it nor not light it)?

There are no relevant differences between the two scenarios. The courtiers who interpret the signal sent from the signalman in accordance with its ought-to-send conditions are expecting too much from him. They have imposed obligations on him that he cannot live up to. Glut and gap theorists, who think that each sentence is true iff its ought-to-be-true conditions are satisfied, are expecting too much of language. The language community has imposed obligations on the language that it cannot live up to.

2 METHODOLOGY

Above I have applied Priest's (1994) own Principle of Uniform Solution: similar paradoxes should be given similar solutions. It is clear that the solution in the case of scenario 2 is that Kuvata fails to represent the facts as they are because the set of sent signals must include some that misrepresent

the facts (relative to the interpretation of the signals by someone who fails to realise the problem with the instructions given to the signaller), and I therefore concluded that English fails to represent the facts as they are because the set of true sentences must include some that misrepresent the facts (relative to the interpretation of English sentences by someone who fails to realise the problem with the instructions we give to each other in the form of language conventions).

There is an obvious objection: the Principle of Uniform Solution should not be applied to the pair consisting of scenario 1 and scenario 2 because the latter is not a paradox! Answer: Excluding that particular pair for that particular reason would run counter to common methodology for problem solving. Why is scenario 2 not a real paradox? Because the solution is too obvious. (Only if we had been convinced that Kuvata would have to be perfect, by necessity, would we ever have considered scenario 2 to be truly paradoxical—but as soon as one is confronted with scenario 2 it becomes clear that he cannot be perfect.) But not applying the Principle of Uniform Solution to the pair for that reason amounts to ignoring insight from a solution to a simple version of a problem when trying to deal with a complicated version. When someone faces a difficult problem and is in a position to benefit from experience with a similar but simpler problem, it is rarely a good idea to dismiss that experience with the words “I was too successful when solving the simpler problem. I should only transfer experience from other problems I never managed to solve.”

That the two scenarios give rise to similar paradoxes, so that the Principle of Uniform Solution applies, even follows from Priest’s (1994, 28; 1995, 47) own analysis of what it takes to be similar, in the relevant respect, to the liar paradox: namely that the paradox in question fits into the following schema.

There are two properties φ and ψ and a (possibly partial) function δ such that

1. $\Omega = \{y \mid \varphi(y)\}$ exists, $\psi(\Omega)$, and
2. if x is a subset of Ω such that $\psi(x)$, then
 - (a) $\delta(x) \notin x$ and
 - (b) $\delta(x) \in \Omega$

In order to fit scenario 2 into the schema, we let y range over instants of time and x range over sets of instants of time, and we let δ map sets of instants of time to instants of time. Further, for any condition ϕ , let $\langle \phi \rangle$ be the unique instant of time t such that Kuvata has been instructed by the court to light the fire at t iff ϕ is satisfied (if there is such a t ; otherwise,

it is not defined). Then, we can specify the two properties and the partial function needed:

- $\varphi(y)$: y is an instant of time when a fire is lit by Kuvata
- $\delta(x) = \tau$ where $\tau = \langle \tau \notin x \rangle$ (if there is such an instant τ ; otherwise, it is not defined)
- $\psi(x)$: the court has given instructions that imply that $\delta(x)$ is defined²

The first half of condition 1 is satisfied, because $\Omega = \{y \mid \varphi(y)\}$ is the set of instants of time when a fire is lit by Kuvata, so it exists. The second half is as well: the court has given instructions that imply that sundown is the unique instant of time t such that the court has instructed Kuvata to light a fire at t iff $t \notin \Omega$, so $\psi(\Omega)$.

To show that condition 2 is satisfied for the liar paradox, Priest makes use of the T-schema. The analogue here is a principle we can name *Perfection*: for any condition ϕ for which $\langle \phi \rangle$ is defined, $\langle \phi \rangle \in \Omega$ iff ϕ is satisfied (that is, Kuvata is perfect: he lights the fire exactly when the ought-to-light condition is satisfied).

Now, assume that a is a subset of Ω such that $\psi(a)$. Then we have

$$\begin{aligned}
 \delta(a) \in a &\Rightarrow \langle \tau \notin a \rangle \in a \\
 &\Rightarrow \langle \tau \notin a \rangle \in \Omega \\
 &\Rightarrow \tau \notin a && \text{(by Perfection)} \\
 &\Rightarrow \delta(a) \notin a
 \end{aligned}$$

Hence, $\delta(a) \notin a$, so condition 2a is satisfied. Moreover, it follows that $\tau \notin a$, and hence by Perfection that $\langle \tau \notin a \rangle \in \Omega$, i.e., $\delta(a) \in \Omega$, so 2b is also satisfied.

Since $\psi(\Omega)$, a contradiction follows: $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$.

Of course, Perfection is not true, but as Priest (2010, 360–361) writes, “[i]t is not required that the arguments entailing the conditions be sound.” The principles used in them just have to be *prima facie* plausible, and Perfection is plausible, *until* one is confronted with scenario 2 or something similar, just like the T-schema is plausible, *until* one is confronted with scenario 1 or something similar.

I should mention that, by invoking the Principle of Uniform *Solution*, I do not mean to imply that this paper contains a full solution to the liar paradox.

²In the case of the liar paradox, y ranges over sentences, x ranges over sets of sentences, $\varphi(y)$ means that y is true, $\psi(x)$ means that x is definable, and δ is a function defined by diagonalisation in such a way that if x is definable then $\delta(x) = \alpha$ where $\alpha = \langle \alpha \notin x \rangle$, where $\langle \phi \rangle$ now is a sentence that is true iff ϕ is satisfied.

It does not. I am merely trying to say something about possible solutions. On the one hand, I have ruled out two extreme positions concerning the liar. And on the other, the point about language being potentially misleading should lower our expectations regarding any other potential solution. Most attempted solutions try to convince us of two things: namely, that truth values are distributed over the sentences of a language that contains the liar in some specific way, and that such distribution is entirely natural and satisfying. My point is that we have to acknowledge that no distribution of truth values is entirely natural and satisfying, and so neither is the actual distribution, whatever it happens to be. We have demanded so much of language that something was bound to go wrong.

Let me elaborate a bit. My claim that truth misrepresents the facts as they are may seem as baffling as the liar paradox itself. How could *truth* possibly come apart from what correctly represents the facts?! Well, it only does relative to a certain interpretation of the true sentences, and the actual truth conditions of some sentences must be different from what we think they are, when we interpret the true sentences that way. Kuvata only misrepresents the facts relative to a certain way of interpreting his signals. The actual conditions under which he lights a fire are different from what the courtiers think they are, when they interpret his signals in such a way that the facts are misrepresented to them. Just to mention one of many possibilities, Kuvata might default to not lighting the fire when he receives the kind of paradoxical instructions of scenario 2. That is, when the court gives an order of the form *Kuvata lights the fire at t iff the fire is not lit at Mount m at t*, the actual condition might be that *Kuvata lights the fire at t iff the fire is not lit at Mount m at t and Mount m is different from Mount Suomalainen*. Similarly, actual truth conditions of paradoxical sentences must come apart, in some way, from what sentences carry on their sleeves. So a detailed theory about the distribution of truth values should not be admonished for implying this necessity.

In addition to the formal demonstration that scenario 2 satisfies Priest's criterion for being relevantly similar to scenario 1, I would like to explain more informally why I think that scenario 2 is particularly well suited to teach us something about scenario 1. That is because there is an ideal combination of similarities and differences between the two. Among the similarities is, first, the fact that they both concern a language: the sentences in scenario 2 are the potential acts of lighting a fire at a specific time and place and the potential acts of refraining from doing so. Second, both languages contain self-referential sentences. And third, they are both systems devised for the purpose of communication by fallible humans. The difference is that the paradoxical sentence in scenario 1 makes reference to *truth*, which is a difficult concept to understand, historically causing philosophers to formulate and defend almost any imaginable position regarding it, while the

paradoxical sentences in scenario 2 instead refers to a fire being lit, which could hardly be more concrete.³ This concreteness makes the solution to the paradox of scenario 2 obvious. As this is the only significant difference between the two scenarios, there is only a degree-of-obviousness difference between the solutions and not a substantial one: in both cases the humanly created systems are prevented from being perfect by the basic metaphysical principle that states of affairs either obtain or not.

3 A CLARIFICATION

The opponents I have argued against above are the gap theorist who thinks that one and the same sentence can both fail to be true and fail to be not true⁴ and the glut theorist who thinks that one and the same sentence can be both true and not true. I have no beef with anyone who simply claims that some sentence is not true while the same sentence prefaced with, e.g., “It is not the case that” is also not true, or, dually, claims that both sentences are true. The kind of argument used in this paper cannot be employed to show that someone of that persuasion is wrong, as the following scenario demonstrates.

Scenario 3: One year, Kuvata receives the following instructions: “Each day in the coming year, if the signalman on Mount Suomalainen does not light a fire at sundown—and only if he does not—you must light a fire at sundown. Also, if the signalman on Mount Suomalainen does light a fire at sundown—and only if he does—you must light a fire one sandglass after sundown.”

Just as in scenario 2, Kuvata either lights the fire at sundown or he doesn’t, and whether it’s one or the other, that signal will be misleading. The point of this new scenario is that one sandglass after sundown, Kuvata can light a fire or refrain from doing so, and he is not forced to do the opposite of what he did at sundown even though the ought-to-light conditions for the two instants are opposite. (In fact, if Kuvata wants to minimise the number of misleading signals he sends, after he has been forced to abandon the ideal of never sending misleading signals, he will light the fire one sandglass after sundown if he lit the fire at sundown, and not if not.)

³It seems to me that Eldridge-Smith (Eldridge-Smith and Eldridge-Smith 2010; Eldridge-Smith 2011) attempted to accomplish something similar when he compared the liar paradox to the paradox ensuing from Pinocchio saying “My nose will be growing”. A growing nose is also more concrete than *truth*. But I think that he failed because the world of Pinocchio is so outlandish that it is not immediately clear to us what would happen in it: is the magic so strong that it can defy a basic metaphysical principles? And if not, then what exactly happens when it runs up against it? Creating a system of communication based on fires, on the other hand, requires no magic and is something we could easily do, and we have no difficulty imagining how it would work and how it would fail.

⁴Whether or not they would be willing to put the thesis in those terms; see footnote 1.

This has consequences. If we combine what I have argued for (in section 1) with what I have made explicit that I will not rule out (in this section), it follows that λ cannot be both true and not true, but that the glut theorist who claims as much by asserting both “ λ is true” and “ λ is not true” might be using only true sentences to do so (and vice versa for the gap theorist).⁵

However, that just serves to underscore my point: true sentences can be misleadingly true (or non-true sentences can be misleadingly non-true). I can only hope I haven’t *used* too many such sentences in this attempt to communicate about a communication problem—and, to the extent that I have, that the reader is not as easily misled as the courtiers who interpret all received signals strictly in accordance with the instructions given to the signalmen.

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⁵Thus, my position has similarities to (but is not identical with) Mares’s (2004), who rejects *metaphysical dialetheism* while accepting the possibility of *semantic dialetheism*; and to Tahko’s (2009), who tries to formulate and defend a metaphysical version of the law of non-contradiction.